A New Twist to the Miners’ Puzzle
Partly based on joint work with Jeroen Groenendijk and Floris Roelofsen

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SPE7
A New Twist to the Miners’ Puzzle

Martin Aher

The Puzzle

The Story

Kratzer semantics
Conditionals

The Analysis

Missing premises

Suppositional Inquisitive Semantics

The language
Propositions
The Recursive definitions
Comparing support
Supposability check

Solution

Back to the miners’ puzzle
Desideratum 1
Desideratum 2
Desideratum 3
### The Facts

- There are two mine shafts.
- Blocking the **correct** mine shaft saves all miners.
- Blocking the **wrong** mine shaft kills all miners.
- Blocking **neither** mine shaft kills one miner.

### Desideratum 1

(1) We ought to block neither shaft.  

\[ \Box (\neg p' \land \neg q') \]
The facts

- There are two mine shafts.
- Blocking the correct mine shaft saves all miners.
- Blocking the wrong mine shaft kills all miners.
- Blocking neither mine shaft kills one miner.

Desideratum 1

(1) We ought to block neither shaft. \[ \boxed{\neg p' \land \neg q'} \]
Kolodny and MacFarlane

**Premises**

<table>
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<th>No.</th>
<th>Statement</th>
<th>Symbolization</th>
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<td>(2)</td>
<td>a. The miners are in shaft A or B.</td>
<td>$p \lor q$</td>
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<td>b. If the miners are in shaft A, we ought to block shaft A.</td>
<td>$p \rightarrow \Box p'$</td>
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<td></td>
<td>c. If the miners are in shaft B, we ought to block shaft B.</td>
<td>$q \rightarrow \Box q'$</td>
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**The Problem**

1. $(p \lor q) \land (p \rightarrow \Box p') \land (q \rightarrow \Box q')$
   
   does not entail $(\not \models)$

2. $\Box (\neg p' \land \neg q')$
Kolodny and MacFarlane

Premises

(2)  a. The miners are in shaft A or B. \( p \lor q \)

b. If the miners are in shaft A, we ought to block shaft A. \( p \to \Box p' \)

c. If the miners are in shaft B, we ought to block shaft B. \( q \to \Box q' \)

The problem

1. \((p \lor q) \land (p \to \Box p') \land (q \to \Box q')\) does not entail \((\nvdash)\)

2. \(\Box (\neg p' \land \neg q')\)
**Kratzer Semantics**

### Modal Base

\[ \begin{align*}
&\text{\textbf{\( pp' \)}} & &\text{\textbf{\( pq' \)}} & &\text{\textbf{\( p(p'q') \)}} \\
&\text{\textbf{\( qq' \)}} & &\text{\textbf{\( qp' \)}} & &\text{\textbf{\( q(p'q') \)}}
\end{align*} \]

### The Ordering

\[ pp', qq' \succ p(p'q'), \quad q(p'q') \succ pq', qp' \]

### Characterization of Obligation:

\[ \Box \varphi \text{ holds when the best worlds are } \varphi \text{ worlds.} \]
Kratzer Semantics

Modal base

- \( pp' \)
- \( qq' \)
- \( pq' \)
- \( qp' \)
- \( p(p'q') \)
- \( q(p'q') \)

The ordering

\[ pp', qq' > p(p'q'), q(p'q') > pq', qp' \]

Characterization of obligation:

\[ \Box \phi \] holds when the best worlds are \( \phi \) worlds.
# A New Twist to the Miners’ Puzzle

## Martin Aher

# The Puzzle

The Story
- Kratzer semantics
- Conditionals

The Analysis
- Missing premises
- Suppositional Inquisitive Semantics
- The language
- Propositions
- The Recursive definitions
- Comparing support
- Supposability check

Solution
- Back to the miners’ puzzle
- Desideratum 1
- Desideratum 2
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## Kratzer Semantics

### Modal Base

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<tr>
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<tr>
<td></td>
<td>(qq')</td>
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### The Ordering

\(pp', qq' > p(p'q'), q(p'q') > pq', qp'\)

### Characterization of Obligation:

\(\Box \varphi\) holds when the best worlds are \(\varphi\) worlds.
MORE TO BE EXPLAINED

**CONDITIONALS**

(3)  

a. If the miners are in shaft A, we ought to block shaft A.  

\[ p \rightarrow \Diamond p' \]

b. If the miners are in shaft B, we ought to block shaft B.  

\[ q \rightarrow \Diamond q' \]

**DESIDERATUM 2:**

\[ \Diamond p' \lor \Diamond q' \] does not hold.
**Implicit Arguments**

**Kratzer [MS]: Assumption of Ignorance**

(4)  
a. Given that we don’t know where the miners are, if the miners are in shaft A, we ought to block shaft A.

b. Given that we don’t know where the miners are, if the miners are in shaft B, we ought to block shaft B

**Cariani, Kaufmann, Schwager [2012]**

"If the miners are in shaft A, we (still) ought to block neither shaft, for their being in shaft A doesn’t mean that we know where they are. Indeed, no matter where the miners are, we ought to block neither shaft."
**Implicit Arguments**

**Kratzer [ms]: Assumption of Ignorance**

(4)  

a. Given that we don’t know where the miners are, if the miners are in shaft A, we ought to block shaft A.

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**Cariani, Kaufmann, Schwager [2012]**

"If the miners are in shaft A, we (still) ought to block neither shaft, for their being in shaft A doesn’t mean that we know where they are. Indeed, no matter where the miners are, we ought to block neither shaft."
The conditionals are not always acceptable

Kratzer: implicit that we will learn that the antecedent is the case

(5)  
a. If the miners are in shaft A, we ought to get sandbags right away and block it.  
b. If the miners are in shaft A, we ought to act fast and block it before the miners suffocate.  
c. If the miners are in shaft A, let’s get sandbags and block it!
**Recap**

**Desiderata:**

1: $\Box (\neg p' \land \neg q')$ holds.

2: $\Box p' \lor \Box q'$ does not hold.

3: Explanation why the conditionals are not always acceptable.

**Next**

Reanalyzing the premises.
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Innocuous premises

Restriction on actions

(6) We cannot block both shafts.
\[ \neg(p' \land q') \]

Restriction on possibilities

(7) The miners are not in both shafts.
\[ \neg(p \land q) \]
Gambling with lives is immoral

(8) a. If it is possible that the miners are in shaft A, then we ought not to block shaft B. \( \Diamond p \rightarrow \Box \neg q' \)

b. If it is possible that the miners are in shaft B, then we ought not to block shaft A. \( \Diamond q \rightarrow \Box \neg p' \)

Intent

When \( \Diamond p \land \Diamond q \) holds then \( \Box (\neg p' \land \neg q') \) holds as well.
**Making more rules explicit**

**Gambling with lives is immoral**

(8)  
- a. If it is possible that the miners are in shaft A, then we ought not to block shaft B. $\Diamond p \rightarrow \Box \neg q'$

- b. If it is possible that the miners are in shaft B, then we ought not to block shaft A. $\Diamond q \rightarrow \Box \neg p'$

**Intent**

When $\Diamond p \land \Diamond q$ holds then $\Box (\neg p' \land \neg q')$ holds as well.
**Rejecting the Original Premises**

**Implicit: We need to know that p holds.**

(9) a. If the miners must be in shaft A, we ought to block shaft A. $\Box p \rightarrow \Diamond p'$

b. If the miners must be in shaft B, we ought to block shaft B. $\Box q \rightarrow \Diamond q'$

**Intent**

- When $\Diamond \neg p$ holds, $\Diamond p'$ does not hold.
- When $\Diamond \neg q$ holds, $\Diamond q'$ does not hold.

**Problem in Kratzer Semantics**

- When $\Box p$ does not hold, (9-a) vacuously holds.
- When $\Box q$ does not hold, (9-b) vacuously holds.
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Rejecting the original premises

Implicit: We need to know that p holds.

(9) a. If the miners must be in shaft A, we ought to block shaft A. □p → □p’

b. If the miners must be in shaft B, we ought to block shaft B. □q → □q’

Intent

▶ When ◊¬p holds, □p’ does not hold.
▶ When ◊¬q holds, □q’ does not hold.

Problem in Kratzer semantics

▶ When □p does not hold, (9-a) vacuously holds.
▶ When □q does not hold, (9-b) vacuously holds.
**Rejecting the Original Premises**

**Implicit: We need to know that \( p \) holds.**

(9) a. If the miners must be in shaft A, we ought to block shaft A. \( \Box p \rightarrow \Box p' \)

b. If the miners must be in shaft B, we ought to block shaft B. \( \Box q \rightarrow \Box q' \)

**Intent**

- When \( \Diamond \neg p \) holds, \( \Box p' \) does not hold.
- When \( \Diamond \neg q \) holds, \( \Box q' \) does not hold.

**Problem in Kratzer Semantics**

- When \( \Box p \) does not hold, (9-a) vacuously holds.
- When \( \Box q \) does not hold, (9-b) vacuously holds.

**Implicit**: we need to know that \( p \) holds.
Suppositional Inquisitive Semantics
Aims

Characteristics

- The semantics specifies when supposition failure occurs, for example when $s = \emptyset$.
- Modified Andersonian Deontic modals are raised to a suppositional semantics.
- Implication, suppositionally deontic may and epistemic might are structurally related.
- Epistemic might is a supposability check (similarly to Veltman’s might as a consistency check.)
- Deontic and epistemic may and must are duals.
# Logical Language

## A Language of Propositional Logic

- Connectives $\neg, \land, \rightarrow$
- Epistemic modal possibility operator $\Diamond$
- Deontic modal permission operator $\lozenge$

## Introduced by Definition:

- $\Box \varphi := \neg \Diamond \neg \varphi$
- $\lozenge \varphi := \neg \lozenge \neg \varphi$
Information states

Worlds and rulings

- A world \( w \) is a valuation function such that for every atomic sentence \( p \): \( w(p) = 1 \) (true) or \( w(p) = 0 \) (false).

\( \omega \) refers to the set of all possible worlds.

- A ruling \( r \) is a violation function such that for every world \( w \in \omega \): \( r(w) = 1 \) (no violation) or \( r(w) = 0 \) (violation).

\( \rho \) refers to the set of all possible rulings.
Global structure of the semantics

Recursive definition of three basic semantic relations:

1. $s \models^+ \varphi$: state $s$ supports $\varphi$
2. $s \models^- \varphi$: state $s$ rejects $\varphi$
3. $s \models^\circ \varphi$: state $s$ dismisses a supposition of $\varphi$

The proposition expressed by $\varphi$, $[\varphi]$, is determined by:

$$[\varphi] = \langle [\varphi]^+, [\varphi]^-, [\varphi]^\circ \rangle$$

where

$[\varphi]^+$ denotes $\{ s \subseteq \omega | s \models^+ \varphi \}$, and similarly for $[\varphi]^-$ and $[\varphi]^\circ$
PROPOSITIONS AND DISMISSAL

A PROPOSITION IS A TRIPLE $\mathcal{P} = \langle P^+, P^-, P^\circ \rangle$ WHERE:

- $P^\circ$ is a downward closed set of states:
  if $s \in P^\circ$ and $t \subset s$, then $t \in P^\circ$
- $P^+$ and $P^-$ are not downward closed.
- $P^+$ and $P^-$ are mutually exclusive: $(P^+ \cap P^-) = \emptyset$
- $P^+$ and $P^-$ are consistent: $\emptyset \notin (P^+ \cap P^-)$
- If a state has no substate that supports or rejects $P$, then a state suppositionally dismisses $P$:
  if $\forall t \subseteq s : t \notin (P^+ \cup P^-)$, then $s \in P^\circ$

CRUCIAL FACT:

Any proposition is suppositionally dismissed by the inconsistent state:
for all $\mathcal{P} : \emptyset \in P^\circ$
A proposition is a triple $\mathcal{P} = \langle \mathcal{P}^+, \mathcal{P}^-, \mathcal{P}^\circ \rangle$ where:

- $\mathcal{P}^\circ$ is a downward closed set of states:
  
  if $s \in \mathcal{P}^\circ$ and $t \subseteq s$, then $t \in \mathcal{P}^\circ$

- $\mathcal{P}^+$ and $\mathcal{P}^-$ are not downward closed.

- $\mathcal{P}^+$ and $\mathcal{P}^-$ are mutually exclusive: $(\mathcal{P}^+ \cap \mathcal{P}^-) = \emptyset$

- $\mathcal{P}^+$ and $\mathcal{P}^-$ are consistent: $\emptyset \notin (\mathcal{P}^+ \cap \mathcal{P}^-)$

- If a state has no substate that supports or rejects $\mathcal{P}$, then a state suppositionally dismisses $\mathcal{P}$:
  
  if $\forall t \subseteq s: t \notin (\mathcal{P}^+ \cup \mathcal{P}^-)$, then $s \in \mathcal{P}^\circ$

**Crucial fact:**

Any proposition is suppositionally dismissed by the inconsistent state:

for all $\mathcal{P}$: $\emptyset \in \mathcal{P}^\circ$
Supposability of Alternatives

Alternatives for a Proposition

\[ \text{alt}(\mathcal{P}) := \{ s \in \mathcal{P}^+ | \text{there is no } t \in \mathcal{P}^+ \text{ such that } t \supseteq s \} \]

Supposability

- Let \( \alpha \in \text{alt}(\mathcal{P}) \) (which implies that \( \alpha \in \mathcal{P}^+ \))
- Then we say that \( \alpha \) is supposable in \( s \), notation \( s \triangleleft \alpha \),
  iff \( \forall t: \text{if } \alpha \supseteq t \supseteq (\alpha \cap s), \text{ then } t \in \mathcal{P}^+ \)

Supposability Implies Consistency

- \( s \triangleleft \alpha \) implies that \( (\alpha \cap s) \neq \emptyset \)
Deontic suppositional inquisitive semantics

Ordinary atomic sentences

- \( s \models^+ p \) iff \( s \neq \emptyset \) and \( \forall w \in \text{worlds}(s) : w(p) = 1 \)
- \( s \models^- p \) iff \( s \neq \emptyset \) and \( \forall w \in \text{worlds}(s) : w(p) = 0 \)
- \( s \models^\circ p \) iff \( s = \emptyset \)

The deontic predicate OK

- \( s \models^+ \text{OK} \) iff \( s \neq \emptyset \) and \( \forall w \in \text{worlds}(s) \) and \( \forall r \in \text{rulings}(s) : r(w) = 1 \)
- \( s \models^- \text{OK} \) iff \( s \neq \emptyset \) and \( \forall w \in \text{worlds}(s) \) and \( \forall r \in \text{rulings}(s) : r(w) = 0 \)
- \( s \models^\circ \text{OK} \) iff \( s = \emptyset \)
### Choosing Directions in Deontic States

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<th>$S_1$</th>
<th>$W_1$</th>
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## Negation, Disjunction, Conjunction

### Negation

- $s \models^+ \neg \varphi$ iff $s \models^\varphi$
- $s \models^\varphi$ iff $s \models^{+} \varphi$
- $s \models^\varphi$ iff $s \models^{\circ} \varphi$

### Disjunction

- $s \models^+ \varphi \lor \psi$ iff $s \models^+ \varphi$ or $s \models^+ \psi$
- $s \models^- \varphi \lor \psi$ iff $s \models^- \varphi$ and $s \models^- \psi$
- $s \models^\varphi \lor \psi$ iff $s \models^\varphi$ or $s \models^\varphi$

### Conjunction

- $s \models^+ \varphi \land \psi$ iff $s \models^+ \varphi$ and $s \models^+ \psi$
- $s \models^- \varphi \land \psi$ iff $s \models^- \varphi$ or $s \models^- \psi$
- $s \models^\varphi \land \psi$ iff $s \models^\varphi$ or $s \models^\varphi$
Clauses for Implication

- $s \models^+ \varphi \rightarrow \psi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and $\forall \alpha \in \text{ALT}[\varphi]^+$:
  1. $s \triangleleft \alpha$, and
  2. $\alpha \cap s \models^+ \psi$

- $s \models^- \varphi \rightarrow \psi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and $\exists \alpha \in \text{ALT}[\varphi]^+$:
  1. $s \triangleleft \alpha$, and
  2. $\alpha \cap s \models^- \psi$

- $s \models^\circ \varphi \rightarrow \psi$ iff $\text{ALT}[\varphi]^+ = \emptyset$ or $\exists \alpha \in \text{ALT}[\varphi]^+$:
  1. $s \not\triangleleft \alpha$, or
  2. $\alpha \cap s \models^\circ \psi$

Example

(10) If Mary sings, Sue will dance. $p \rightarrow q$

a. No, if Mary sings, Sue will not dance. $p \rightarrow \neg q$

b. Well, Mary won’t sing. $\neg p$
Deontic Modals
### Deontic *may*

- \( s \models^+ \Diamond \varphi \) iff \( \text{ALT}[\varphi]^+ \neq \emptyset \) and \( \forall \alpha \in \text{ALT}[\varphi]^+ : \)
  1. \( s \triangleleft \alpha \), and
  2. \( \alpha \cap s \models^+ \text{OK} \)

- \( s \models^- \Diamond \varphi \) iff \( \text{ALT}[\varphi]^+ \neq \emptyset \) and \( \forall \alpha \in \text{ALT}[\varphi]^+ : \)
  1. \( s \triangleleft \alpha \), and
  2. \( \alpha \cap s \models^- \text{OK} \)

- \( s \models^\circ \Diamond \varphi \) iff \( \text{ALT}[\varphi]^+ = \emptyset \) or \( \exists \alpha \in \text{ALT}[\varphi]^+ : \)
  1. \( s \vartriangleleft \alpha \)
### Obvious Difference

- The one difference is that the ‘consequent’ of *may* is not an arbitrary formula, but the **deontic predicate** OK.

  \[ s \models^+ \bigcirc \varphi \iff s \models^+ \varphi \rightarrow \text{OK} \]

- The deontic predicate OK is atomic, so it is **not suppositional**.

- \[ s \models^+ (\varphi \lor \psi) \rightarrow \text{OK} \iff s \models^+ \varphi \rightarrow \text{OK} \land \psi \rightarrow \text{OK}, \text{ so} \]

  \[ s \models^+ \bigcirc (\varphi \lor \psi) \iff s \models^+ \bigcirc \varphi \land \bigcirc \psi \]
Deontic free choice

Free choice

(11)  
\begin{align*}
  &a. \quad \text{A country may establish a research center or a laboratory.} \\
  &b. \quad \Diamond (p \lor q)
\end{align*}

Support clause of $\Diamond \varphi$

\[ s \models^+ \Diamond \varphi \iff \text{ALT}[\varphi]^+ \neq \emptyset \quad \text{and} \quad \forall \alpha \in \text{ALT}[\varphi]^+: \]
\[ 1. \quad s \prec \alpha, \quad \text{and} \]
\[ 2. \quad \alpha \cap s \models^+ \text{OK} \]

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Table 1: $s_1 \models^+ \Diamond (p \lor q)$
Comparing deontic *may* and implication

**Crucial Difference**

- $s \models {\nabla} \varphi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and $\forall \alpha \in \text{ALT}[\varphi]^+ :$
  
  1. $s < \alpha$, and
  
  2. $\alpha \cap s \models {\neg} \text{OK}$

- $s \models \varphi \rightarrow \psi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and $\exists \alpha \in \text{ALT}[\varphi]^+ :$
  
  1. $s < \alpha$, and
  
  2. $\alpha \cap s \models {\neg} \psi$

**Implications with Support-Inquisitive Antecedents**

(12) If Sue sings or Mary dances, then Pete will play the Piano.

a. No, if Sue sings, Pete will *not* play the Piano.

b. No, if Mary dances, Pete will *not* play the Piano.
Comparing deontic *may* and implication

**Crucial difference**

- \( s \models \neg \Diamond \varphi \) iff \( \text{ALT}[\varphi]^+ \neq \emptyset \) and \( \forall \alpha \in \text{ALT}[\varphi]^+ : \)
  1. \( s \triangleright \alpha \), and
  2. \( \alpha \cap s \models \neg \text{OK} \)

- \( s \models \neg \varphi \rightarrow \psi \) iff \( \text{ALT}[\varphi]^+ \neq \emptyset \) and \( \exists \alpha \in \text{ALT}[\varphi]^+ : \)
  1. \( s \triangleright \alpha \), and
  2. \( \alpha \cap s \models \neg \psi \)

**Implications with support-inquisitive antecedents**

(12) If Sue sings or Mary dances, then Pete will play the Piano.

  a. No, if Sue sings, Pete will *not* play the Piano.
  b. No, if Mary dances, Pete will *not* play the Piano.
Deontic Free choice

Negating free choice

(13)  a. A country may not establish a research center or a laboratory.
     b. $\neg \Diamond (p \lor q)$

Reduced rejection clause of $\Diamond \varphi$

$s \models \neg \Diamond \varphi$ iff $\text{ALT}[\varphi]^+ \neq \emptyset$ and $\forall \alpha \in \text{ALT}[\varphi]^+: \alpha \cap s \models \neg \text{OK}$

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<tbody>
<tr>
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<td>10</td>
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</tbody>
</table>

Table 2: $s_1 \models ^+ \neg \Diamond (p \lor q)$
**Comparing deontic **may** AND IMPLICATION**

<table>
<thead>
<tr>
<th><strong>Difference disappears, when φ is not support-inquisitive</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>- If φ is <strong>not</strong> support-inquisitive:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>[ s \models ^- \lozenge \varphi \iff s \models ^- \varphi \rightarrow \text{OK} ]</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Taking the difference into account:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. [ s \models ^- \lozenge \varphi \iff s \models ^+ \varphi \rightarrow \neg\text{OK} ]</td>
</tr>
<tr>
<td>2. [ s \models ^+ \neg\lozenge \varphi \iff s \models ^+ \varphi \rightarrow \neg\text{OK} ]</td>
</tr>
<tr>
<td>3. [ s \models ^+ \lozenge \varphi \iff s \models ^+ \neg \varphi \rightarrow \neg\text{OK} ]</td>
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</tbody>
</table>
**Deontic Free choice**

**Dismissing a free choice prohibition**

(14)  

<p>| | | | |</p>
<table>
<thead>
<tr>
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<tr>
<td>a.</td>
<td>A country may not establish a research center or a laboratory.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>$\neg \Box (p \lor q)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Reduced dismissal clause of $\Box \varphi$**

$$s \models ^{\circ} \Box \varphi \iff \text{alt}[\varphi]^+ = \emptyset \lor \exists \alpha \in \text{alt}[\varphi]^+ : \alpha \cap s = \emptyset$$

**Dismissal**

(15)  

<p>| | | | |</p>
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<thead>
<tr>
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<tbody>
<tr>
<td>a.</td>
<td>Well, no country will establish a research center.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>$\neg p$</td>
<td></td>
<td></td>
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</tbody>
</table>

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<td>01</td>
<td>00</td>
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</tbody>
</table>
## Deontic Free choice

### Dismissing a free choice prohibition

(16)  
\[ \text{a. A country may not establish a research center or a laboratory.} \]
\[ \text{b. } \neg \Diamond (p \lor q) \]

### Reduced dismissal clause of $\Diamond \varphi$

\[ s \models \Diamond \varphi \text{ iff } \text{alt}[\varphi]^+ = \emptyset \text{ or } \exists \alpha \in \text{alt}[\varphi]^+ : \alpha \cap s = \emptyset \]

### Dismissal

(17)  
\[ \text{a. Well, no country will establish a research center.} \]
\[ \text{b. } \neg p \]

<table>
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</tbody>
</table>
**Conditional Obligation**

**Reduction to Implication**

\[ s \models^{+} \bigvee \varphi \iff s \models^{+} \neg \varphi \rightarrow \neg \text{OK} \]

**Conditional Permission**

(18)  

a. If a country has a laboratory, it **must** establish a research center.  
b. \( p \rightarrow \bigvee q \)  
c. \( p \rightarrow (\neg q \rightarrow \neg \text{OK}) \)  
d. \( (p \land \neg q) \rightarrow \neg \text{OK} \)
Suppositional Inquisitive Semantics

Epistemic modals
**Suppositional Epistemic** *might* and *must*

### Might as a Supposability Check

- In InqS $\diamond \varphi$ can be treated as a supposability check.
- In the most basic cases this boils down to a consistency check, like Veltman’s *might* in update semantics (US).

### Persistence

- For Veltman, $\diamond \varphi$ is a basic example of a non-persistent update.
- InqS epistemic modals are support/reject-persistent modulo suppositional dismissal.
### Necessary relations

#### Suppositionally dismissing supportability

- \( s \models^\otimes \varphi \quad \text{iff} \quad s \models^\circ \varphi \quad \text{and} \quad s \not\models^\prec \varphi \quad \text{and} \quad \forall t \subseteq s: \ t \not\models^+ \varphi. \)

#### For a non-suppositional \( \varphi \)

- \( s \models^\otimes \varphi \quad \text{iff} \quad s = \emptyset. \)

#### Generally

- If \( s \models^\otimes \varphi \), then no alternative for \( \varphi \) is supposable in \( s \).
**Suppositional might: the intuitive idea**

<table>
<thead>
<tr>
<th><strong>◊(\varphi) is a proposal to check the supposability of (\varphi) in (S)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>▶ s supports (◊\varphi) iff</td>
</tr>
<tr>
<td>(A) there is at least one alternative for (\varphi) and</td>
</tr>
<tr>
<td>(B) every alternative for (\varphi) is supposable in (s)</td>
</tr>
<tr>
<td>▶ s rejects (◊\varphi) iff</td>
</tr>
<tr>
<td>(A) s does not suppositionally dismiss supportability of (\varphi) and</td>
</tr>
<tr>
<td>(B) every alternative for (\varphi) is not supposable in (s)</td>
</tr>
<tr>
<td>▶ s dismisses a supposition of (◊\varphi) iff</td>
</tr>
<tr>
<td>(A) there is no alternative for (\varphi) or</td>
</tr>
<tr>
<td>(B) some alternative for (\varphi) is not supposable in (s)</td>
</tr>
</tbody>
</table>
Suppositional *might*: Support and Dismissal

**Support and dismissing a supposition contradict each other**

- **s supports** $\Diamond \varphi$ iff
  
  (A) there is **at least one alternative** for $\varphi$ and
  
  (B) **every alternative** for $\varphi$ is supposable in $s$

- **s dismisses** a supposition of $\Diamond \varphi$ iff
  
  (A) there is **no alternative** for $\varphi$ or
  
  (B) **some alternative** for $\varphi$ is **not supposable** in $s$
**Suppositional might: Rejection and Dismissal**

**Rejection implies suppositional dismissal**

- **s rejects $\Diamond \varphi$ iff**
  
  (A) s does not suppositionally dismiss supportability of $\varphi$ and
  
  (B) every alternative for $\varphi$ is not supposable in s

- **s dismisses a supposition of $\Diamond \varphi$ iff**
  
  (A) there is no alternative for $\varphi$ or
  
  (B) some alternative for $\varphi$ is not supposable in s
**Suppositional might: Persistence**

**Two essential features of the clauses for \( \Diamond \varphi \)**

- Support and dismissing a supposition contradict each other
- Rejection implies dismissal

**Support of might is defeasible**

- It can be the case that \( s \models^+ \Diamond \varphi \) and that it holds for some more informed state \( t \subset s \) that \( t \not\models^+ \Diamond \varphi \), or even \( t \models^- \Diamond \varphi \), but then it will also be the case that \( t \models^- \Diamond \varphi \).
- Despite the fact that suppositional *might* is support-defeasible, it is still support-persistent modulo suppositional dismissal.
**Suppositional might spelled out**

**Epistemic might**

\[ S \models^+ \Diamond \varphi \quad \text{iff} \quad \text{ALT}(\varphi) \neq \emptyset \quad \text{and} \quad \forall \alpha \in \text{ALT}(\varphi): \ S \triangleleft \alpha \]

\[ S \models^- \Diamond \varphi \quad \text{iff} \quad S \nvdash^\otimes \varphi \quad \text{and} \quad \forall \alpha \in \text{ALT}(\varphi): \ S \ntriangleleft \alpha \]

\[ S \models^\circ \Diamond \varphi \quad \text{iff} \quad \text{ALT}(\varphi) = \emptyset \quad \text{or} \quad \exists \alpha \in \text{ALT}(\varphi): \ S \ntriangleleft \alpha \]

**Figure 1:** \( \Diamond \varphi \)
**Derived suppositional must**

<table>
<thead>
<tr>
<th>Must as a non-supposability check</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ϕ is defined as ¬◊¬ϕ</td>
</tr>
<tr>
<td>So, □ϕ is supported in s, when ◊¬ϕ is rejected in s</td>
</tr>
<tr>
<td>◊¬ϕ is a proposal to check for supposability of ¬ϕ in s</td>
</tr>
<tr>
<td>When the check for supposability of ¬ϕ fails in s, ◊¬ϕ is rejected in s and □ϕ is supported in s.</td>
</tr>
<tr>
<td>Conversationally, a speaker proposing □ϕ, invites a responder to suppose that ¬ϕ, in the hope that in her state ¬ϕ is (also) not supposable.</td>
</tr>
</tbody>
</table>
Back to the miners’ puzzle
**Premises:**

(19)  

a. The miners are in in shaft A or B. \( p \lor q \)

b. We cannot block both shafts. \( \neg(p' \land q') \)

c. The miners are not in both shafts. \( \neg(p \land q) \)

d. If the miners must be in shaft A, we ought to block shaft A. \( \Box p \rightarrow \bigvee p' \)

e. If the miners must be in shaft B, we ought to block shaft B. \( \Box q \rightarrow \bigvee q' \)

f. If it is possible that the miners are in shaft A, then we ought not to block shaft B. \( \Diamond p \rightarrow \bigvee \neg q' \)

g. If it is possible that the miners are in shaft B, then we ought not to block shaft A. \( \Diamond q \rightarrow \bigvee \neg p' \)
**The Aim**

**Desiderata:**

1. \( \Box (\neg p' \land \neg q') \) holds.
2. \( \Box p' \lor \Box q' \) does not hold.
3. Explanation why the conditionals are not always acceptable.
We block neither shaft

**Desideratum 1**: $\boxcheck (\neg p' \land \neg q')$ holds.

(20)

a. The miners are in shaft A or B. $p \lor q$
b. We cannot block both shafts. $\neg (p' \land q')$
c. The miners are not in both shafts. $\neg (p \land q)$
d. If it is possible that the miners are in shaft A, then we ought not to block shaft B. $\Diamond p \rightarrow \boxcheck \neg q'$
e. If it is possible that the miners are in shaft B, then we ought not to block shaft A. $\Diamond q \rightarrow \boxcheck \neg p'$

<table>
<thead>
<tr>
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<th>$w_3$</th>
<th>$w_4$</th>
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<th>$w_6$</th>
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</tr>
</tbody>
</table>

**Table 5**: $s \models^+ \boxcheck (\neg p' \land \neg q') \iff s \models^+ (p' \rightarrow \neg OK) \land (q' \rightarrow \neg OK)$
What if we learn that $p$ holds?

**New premises**

(21)  

<table>
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<tr>
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What if we learn that $p$ holds?

**New premises**

(22)  

\begin{align*}
\text{a. } & \text{The miners are in shaft A.} & \quad p \\
\text{b. } & \text{If it is possible that the miners are in shaft B, then we ought not to block shaft A.} & \quad \Diamond q \rightarrow \Box \neg p'
\end{align*}

<table>
<thead>
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What if we learn that $p$ holds?

**New premises**

(23) a. The miners are in shaft A. $p$

b. If it is possible that the miners are in shaft B, then we ought not to block shaft A. $\Diamond q \rightarrow \Box \neg p'$

<table>
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</table>

When we find out that the miners are in Shaft A, the obligation to block neither becomes void.
**We shouldn’t gamble**

### Desideratum 2: \( p' \lor q' \) does not hold.

\[(24)\]

<table>
<thead>
<tr>
<th></th>
<th>a. The miners are in shaft A or B. ( p \lor q )</th>
<th>b. We cannot block both shafts. ( \neg(p' \land q') )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c. The miners are not in both shafts. ( \neg(p \land q) )</td>
<td>d. If the miners <strong>must be</strong> in shaft A, we ought to block shaft A. ( \Box p \rightarrow \Box p' )</td>
</tr>
<tr>
<td></td>
<td>e. If the miners <strong>must be</strong> in shaft B, we ought to block shaft B. ( \Box q \rightarrow \Box q' )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( S_1 )</th>
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<tr>
<td>( r_3 )</td>
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</table>
What if we learn that $p$ holds?

**Premises**

$$\text{(25) \quad \begin{align*}
\text{a. The miners are in shaft A.} & \quad p \\
\text{b. We cannot block both shafts.} & \quad \neg(p' \land q') \\
\text{c. The miners are not in both shafts.} & \quad \neg(p \land q) \\
\text{d. If the miners must be in shaft A, we ought to} & \quad \Box p \rightarrow \lozenge p' \\
\text{block shaft A.} & \\
\text{e. If the miners must be in shaft B, we ought to} & \quad \Box q \rightarrow \lozenge q' \\
\text{block shaft B.} & 
\end{align*}}$$

<table>
<thead>
<tr>
<th>$S_2$</th>
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</tbody>
</table>
DEFEASIBLITY

DESIDERATUM 3

Why aren’t the conditionals always acceptable?

REINTERPRETING THE CONDITIONALS

(26)  

a. If the miners must be in shaft A, we ought to block shaft A.  

□p → □p′

b. If the miners must be in shaft B, we ought to block shaft B.  

□q → □q′

CLEO CONDORAVDI AND SVEN LAUER (A.O.): EPISTEMIC NECESSITY OVER THE ANTECEDENT IN CONDITIONALS

(27) Anankastic: If you want to go to Harlem, you have to take the A-train.
DEFEASIBILITY

DESIDERATUM 3

Why aren’t the conditionals always acceptable?

REINTERPRETING THE CONDITIONALS

(26)  
a. If the miners must be in shaft A, we ought to block shaft A.  \[ \square p \to \bigvee p' \]
b. If the miners must be in shaft B, we ought to block shaft B.  \[ \square q \to \bigvee q' \]

Cleó Condoravdi an Sven Lauer (a.o): epistemic necessity over the antecedent in conditionals

(27)  
Anankastic: If you want to go to Harlem, you have to take the A-train.
THE END (OR IS IT?)

Thank you for listening

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